

**Aristotle, Prior Analytics 1.23**

It is clear then that the ostensive syllogisms are effected by means of the aforesaid figures<sup>1</sup>; these considerations will show that *reductiones ad* also are effected in the same way. For all who effect an argument *per impossibile* infer syllogistically what is false, and prove the original conclusion hypothetically when something impossible results from the assumption of its contradictory; e.g. that the diagonal of the square is incommensurate with the side, because odd numbers are equal to evens if it is supposed to be commensurate. One infers syllogistically that odd numbers come out equal to evens, and one proves hypothetically the incommensurability of the diagonal, since a falsehood results through contradicting this. For this we found to be reasoning *per impossibile*, viz. proving something impossible by means of an hypothesis conceded at the beginning.

**Euclid X - Definitions**

**Def. 1:** Those magnitudes are said to be commensurable which are measured by the same measure, and those incommensurable which cannot have any common measure.

**Euclid X.117**

Suppose  $AC$ , the diagonal of a square, to be commensurable with its side  $AB$ , and let their ratio in its smallest terms be  $a : b$ . Now  $AC^2 : AB^2 = a^2 : b^2$  and  $AC^2 = 2AB^2$ ,  $a^2 = 2b^2$ . Hence  $a^2$ , and therefore  $a$ , is even. Since  $a : b$  is in its lowest terms, it follows that  $b$  is odd. Let  $a = 2c$ . Then  $4c^2 = 2b^2$ , or  $b^2 = 2c^2$ , so that  $b^2$  is even, and therefore  $b$  is even. But  $b$  was shown to be odd, and is therefore odd and even, which is impossible. Therefore  $AC$  cannot be commensurable with  $AB$ .

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<sup>1</sup>In previous sections Aristotle has laid out the different patterns of categorical syllogism. Things like P1. All men are mortal. P2. Socrates is a man. C. Therefore Socrates is mortal.